Maths for Computing
Assignment 5

1. *(5 marks)* Prove that if P and Q are longest paths (of the same length) in a connected graph, then P and Q have at least one vertex in common. Give a detailed proof.

2. *(5 marks)* Prove that if G is a disconnected graph, then $\overline{G}$ is connected.

3. *(5 marks)* Let M be a maximal matching and M' be any matching in a graph G. Prove that $|M| \geq |M'|/2$.

4. *(5 marks)* Prove that a tree always has more leaves than vertices of degree three.

5. *(5 marks)* Prove that Petersen graph does not contain two perfect matchings M and M' such that $M \cap M' = \emptyset$. You can use the results proved in class or tutorials without proving them again. *(Hint: The length of the smallest cycle in Petersen graph is 5)*

6. *(5 marks)* Petersen graph is non-planar. Prove it using Kuratowski's Theorem.